Short Papers

On the Relationship Between TLM and Finite-Difference Methods for Maxwell's Equations

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Abstract—**It is shown that if the expanded node three-dimensional TLM method is operated in a certain way, then it can be numerically equivalent to a finite-difference method. Some comments are made on comparisons between the two approaches.**

I. Introduction

It has been recognized for some time that under particular circumstances, the TLM method can be similar to the finite-difference method, and the relationship has been established in the case of the model for diffusion [1]. In this paper, it is shown that the three-dimensional expanded node model [2]–[4] can also be operated in a way that produces numbers identical to the finite-difference approach, but such a mode of operation for TLM could be regarded as inefficient. A simplification of the analysis also leads to similar conclusions for two-dimensional methods.

II. TLM and Finite-Difference Algorithms

The TLM algorithm may be expressed as

$$
\begin{aligned}
{}_n\boldsymbol{V}^r &= \boldsymbol{S}\,{}_n\boldsymbol{V}^i \\
{}_{n+1}\boldsymbol{V}^i &= \boldsymbol{C}\,{}_n\boldsymbol{V}^r.
\end{aligned}
\tag{1}
$$

$\boldsymbol{V}^i$ and $\boldsymbol{V}^r$ represent the incident and reflected pulses in the entire network at the time interval n. $\boldsymbol{C}$ is a connection matrix such that

$$
\begin{aligned}
C_{ij} &= 1 \quad \text{if port } i \text{ is connected to port } j \\
C_{ij} &= 0 \quad \text{otherwise.}
\end{aligned}
$$

$\boldsymbol{S}$ is a super matrix with the scattering matrices associated with each scattering zone or node as blocks on the diagonal.

It is always possible to express the field quantities ϕ in terms of incident pulses, i.e.

$$\boldsymbol{\phi} = \boldsymbol{q}\boldsymbol{V}^i. \tag{2}$$

Here ϕ is the vector of all the field quantities E_x, E_y, E_z, H_x, H_y, and H_z at all the nodes in the mesh.

It may be possible to express $\boldsymbol{S}$ as

$$\boldsymbol{S} = \boldsymbol{p} \times \boldsymbol{q} + \boldsymbol{r} \tag{3}$$

and indeed this can be done for the shunt and series nodes making up the expanded node mesh.

Thus, for scattering associated with an x-directional shunt node in the mesh [5]

$$\phi = E_x \tag{4}$$

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and

$$\boldsymbol{q} = \frac{2}{Y}[1 \quad 1 \quad 1 \quad 1 \quad Y_0] \tag{5}$$

$$\boldsymbol{p} = [1 \quad 1 \quad 1 \quad 1 \quad 1]^T \tag{6}$$

$$\boldsymbol{r} = -\boldsymbol{I} \tag{7}$$

where I is an identity matrix and where

$$
\begin{aligned}
Y_0 &= 2\varepsilon_r - 4 \\
Y &= 4 + Y_0 + G_0 \\
G_0 &= \sigma \Delta l \sqrt{\frac{\mu_0}{\varepsilon_0}}
\end{aligned}
$$

and where Δl is the space step.

The scattering for a series node is also given in [5]; thus, for an x-directed series node

$$\phi = H_x \tag{8}$$

$$\boldsymbol{q} = \frac{2}{z}\sqrt{\frac{\varepsilon_0}{\mu_0}}[-1 \quad 1 \quad 1 \quad -1 \quad -1] \tag{9}$$

$$\boldsymbol{p} = \sqrt{\frac{\mu_0}{\varepsilon_0}}[1 \quad -1 \quad -1 \quad 1 \quad Z_0]^T \tag{10}$$

$$\boldsymbol{r} = \boldsymbol{I} \tag{11}$$

where

$$
\begin{aligned}
Z_0 &= 2\mu_r - 4 \\
Z &= 4 + Z_0.
\end{aligned}
$$

Following the development in [1], (1)–(3) may be combined

$${}_{n+1}\boldsymbol{\phi} = \boldsymbol{q}\boldsymbol{C}\left(\boldsymbol{p}\,{}_n\boldsymbol{\phi} + \boldsymbol{r}\,{}_n\boldsymbol{V}^i\right).$$

Here, $\boldsymbol{r} \neq 0$, so the TLM method cannot be expressed as a two-time-level algorithm solely in terms of the quantities ϕ.

Thus

$${}_{n+1}\boldsymbol{\phi} = \boldsymbol{qCp}\,{}_n\boldsymbol{\phi} + \boldsymbol{qCrCp}\,{}_{n-1}\boldsymbol{\phi} + \boldsymbol{qCrCr}\,{}_{n-1}\boldsymbol{V}^i. \tag{12}$$

Thus, provided S can be expressed as (3) and provided

$$\boldsymbol{CrCr} = a\boldsymbol{I}$$

where a is a constant, the routine can be expressed solely in terms of ϕ, the field quantities. Under these conditions, the routine becomes

$${}_{n+1}\boldsymbol{\phi} = \boldsymbol{qCp}\,{}_n\boldsymbol{\phi} + \boldsymbol{qCrCp}\,{}_{n-1}\boldsymbol{\phi} + a\,{}_{n-1}\boldsymbol{\phi}. \tag{13}$$

There are many ways of operating the TLM algorithm for the expanded mesh and there is much to explore in (12). One way, instead of having pulses incident at all nodes simultaneously, is to have pulses incident at only shunt nodes at one instant and only at series nodes at one-half time step later.

Suppose that the pulses are incident upon the shunt nodes at time t; then, substituting (4)–(11) in (12) gives one of the

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equations as

$$
\begin{aligned}
&{}_{t+\Delta t/2}H_x\left(i, j+\tfrac{1}{2}, k+\tfrac{1}{2}\right)\\
&\frac{2}{Z}\sqrt{\frac{\varepsilon_0}{\mu_0}}\,\Big\{{}_tE_y\left(i, j+\tfrac{1}{2}, k+1\right)\\
&\qquad -{}_tE_y\left(i, j+\tfrac{1}{2}, k\right)\\
&\qquad +{}_tE_z\left(i, j, k+\tfrac{1}{2}\right)\\
&\qquad -{}_tE_z\left(i, j+1, k+\tfrac{1}{2}\right)\Big\}\\
&+2\,{}_{t-\Delta t/2}H_x\left(i, j+\tfrac{1}{2}, k+\tfrac{1}{2}\right)\\
&-{}_{t-\Delta t/2}H_x\left(i, j+\tfrac{1}{2}, k+\tfrac{1}{2}\right).
\end{aligned}
$$

This is the same as one of the finite-difference equations used by Taflove and Brodwin in [6] in their implementation of Yee's original formulation [7]. The remaining five equations can be derived in a similar way. The two-dimensional method is a simplification of the above, requiring modification of (9)–(11).

III. Comparisons Between TLM and Finite Differences

Great care has to be taken in comparing computer resources for the TLM method with the finite-difference method since much more information is available in the former. In the three-dimensional TLM method operated in the above way, there are three field quantities available at each shunt and series node. This, for example, allows the boundary description for TLM to be twice as fine as for finite differences. In two dimensions, if boundaries are described only at nodes as in finite differences, then incident pulses need only be at alternate nodes at any instant. Thus, an average of two stores for link lines, not four, is required at each node. Alternatively, if the pulses are incident simultaneously at all nodes, then boundaries can exist halfway between nodes as well as at nodes, and the boundary description is again finer than in finite differences. Also, in assessing arithmetical load, it should be recognized that implementation of (2) and (3) requires much less work than a matrix multiplication.

Comparison of the algorithm is interesting, but often there is a balance between computational efficiency and program or data complexity. A much more important difference between TLM and finite difference is that the former is a physical model using transmission lines, while the latter is a mathematical model using differencing. The advantage of TLM is that it provides the engineer with a conceptual model which can be simulated exactly on a digital computer. The comparison should include the modeling philosophy and not just the algorithm details.

Another advantage of the TLM approach is that it can lead to models and algorithms which cannot be readily expressed in terms of the field quantities because the scattering matrix is not easily factorized as in (3). Examples of this are the asymmetrical condensed node or punctual node [8], [9] and the symmetrical condensed node [10], which have the advantage of condensing all six field quantities to one point in space.

In the author's view, the TLM method and the finite difference method complement each other rather than compete with each other. Each leads to a better understanding of the other.

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Approximate Determination of the Characteristic Impedance of the Coaxial System Consisting of an Irregular Outer Conductor and a Circular Inner Conductor

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Abstract **—An elementary formula is presented for the determination of the characteristic impedance of a coaxial transmission line consisting of a circular inner conductor and an irregular outer conductor. In this approach, the irregular outer conductor is replaced by an eccentric circular outer conductor which has the same "shield factor" as an irregular one at the extreme of a small wire, and the same formula is adapted for outer conductors of different shapes by determining values of eccentricity of the equivalent eccentric coaxial lines. The validity of the formula is confirmed by numerical results.**

I. Introduction

Considerable work has been done on the determination of the characteristic impedance of a coaxial transmission line consisting of a circular inner conductor and a noncircular outer conductor [1]–[9]. Elementary formulas for some shapes have been available for small ratios of inner and outer conductors. A formula for

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